USN | | | | | | | | | | 13MCA12

First Semester MCA Degree Examination, June/July 2017
Discrete Mathematical Structures

Time: 3 hrs.   Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Discuss the basic connectives used in logic. And verify that $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology. **(06 Marks)**

b. Let p, q be primitive statements for which the implication $p \rightarrow q$ is false. Determine the truth values for each of the following :
i) $p \vee q$ ii) $\neg p \wedge \neg q$ ii) $q \rightarrow p$ iv) $\neg q \rightarrow \neg p$. **(08 Marks)**

c. Simplify the following statement using the laws of logic : $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)]$. **(06 Marks)**

2 a. Establish to validity of the following argument. $[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$. **(06 Marks)**

b. Provide the reasons for the steps verifying the following argument (here 'a' denotes a specific but arbitrarily chosen element from the universe).

$\forall x[p(x) \rightarrow (q(x) \wedge r(x)]$
$\forall x[p(x) \wedge s(x)]$
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$\therefore \forall x[r(x) \wedge s(x)]$

Steps
$\forall x[p(x) \rightarrow (q(x) \wedge r(x))]$
$\forall x[p(x) \wedge s(x)]$
$p(a) \rightarrow (q(a) \wedge r(a))$
$p(a) \wedge s(a)$
$p(a)$
$q(a) \wedge r(a)$
$r(a)$
$s(a)$
$r(a) \wedge s(a)$
$\therefore \forall x[r(x) \wedge s(x)]$ **(08 Marks)**

c. Provide a proof by contradiction for the following : For every integer n, i) if n^2 is odd then n is odd ii) the square of an even integer is an even integer. **(06 Marks)**

3 a. Explain sets and subsets with examples. For any tow sets A and B, prove that :
$P(A \cap B) = p(A) \cap p(B)$. **(06 Marks)**

b. A survey of 500 television viewers of a sports channel produced the following information : 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not watch any of the three kinds of games :
i) How many viewers in the survey watch all three kinds of games?
ii) How many viewers watch exactly one of the sports? **(08 Marks)**

c. Define permutations and combinations. In how many ways can the letters in DATAGRAM be arranged? How many have all three 'A's together in the arrangements of DATAGRAM? **(06 Marks)**

4 a. Prove that for each n (06 Marks)

b. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. Find a_n in explicit form. (08 Marks)

c. Discuss Euclidian algorithm in detail. (06 Marks)

5 a. State and prove the pigeonhole principle. (06 Marks)

b. Let A = {1, 2, 3, 4, 5, 6, 7} and B = {w, x, y, z). Find the number of onto functions from A to B. (06 Marks)

c. For A = {1, 2, 3, 4, 5} and B = {w, x, y, z} a function f : A $\rightarrow$ B is defined by f = {(1, w), (2, x), (3, x), (4, y), (5, y)}. Find the images of the following subsets of A, under f. A_1 = {1}, A_2 = {1, 2}, A_3 = {1, 2, 3}, A_4 = {2, 3} A_5 = {2, 3, 4, 5}. (08 Marks)

6 a. Let S be a universal set on p(s), define a relation R by (a, B) $\in$ R if and only if A $\subseteq$ B. Prove that R is reflexive, antisymmetric and transitive but not symmetric. (08 Marks)

b. If $A = A_1 \cup A_2 \cup A_3$, where A_1 = {1, 2}, A_2 = {2, 3, 4} and A_3 = {5}, define to relation R on A by xRy if and only if x and y are in the same set A_i, i = 1, 2, 3. Is R an equivalence relation? (06 Marks)

c. If R is a relation on the set A = {1, 2, 3, 4} defined by xRy if x/y, prove that (A, R) is a Poset. Draw its Hasse diagram. (06 Marks)

7 a. Define graph. Prove that if G is an undirected graph, then G is connected iff G has a spanning tree. (06 Marks)

b. Define : i) Euler graph ii) Hamiton graph iii) graph isomorphism iv) cosets. (08 Marks)

c. Define planar graph. Show that K_3^3 is non-planar. (06 Marks)

8 a. Define the following with examples :
i) directed graphs ii) weighted graphs iii) rooted trees. (06 Marks)

b. State and prove the decomposition theorem for chromatic polynomials and find the chromatic polynomial for a cycle of length 4(C_4). (08 Marks)

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z. That occur (in a given sample) with frequencies 20, 28, 4, 17, 12, 7 respectively. (06 Marks)

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